



Non Uniform Sampling

What is Non-Uniform Sampling?

Non-Uniform Sampling (NUS)

is a sampling technique which samples points in indirect dimension(s) of multidimensional experiment *non-uniformly* (= sparsely, randomly)



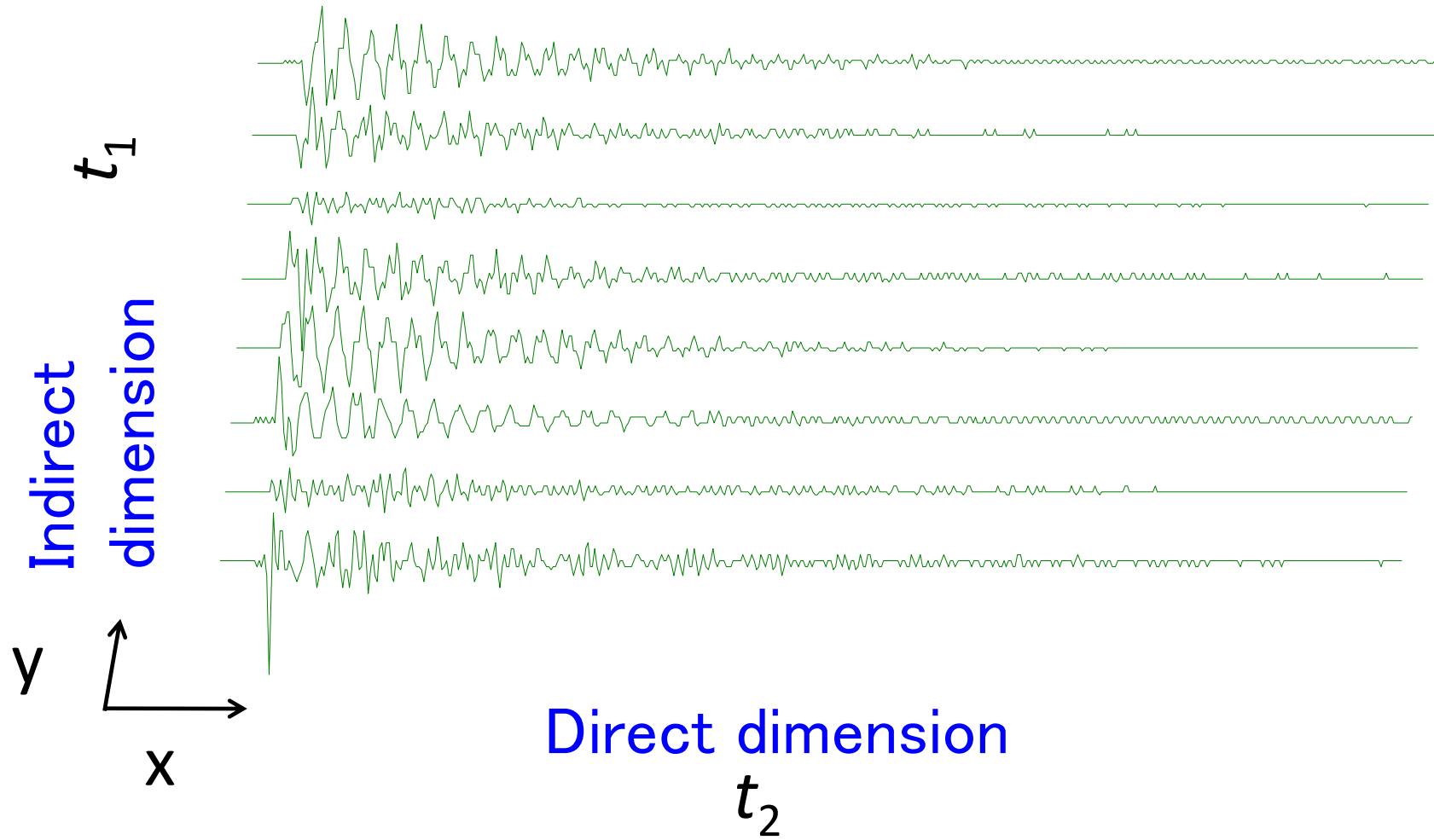
Multidimensional correlation experiments can be made shorter



Caution!

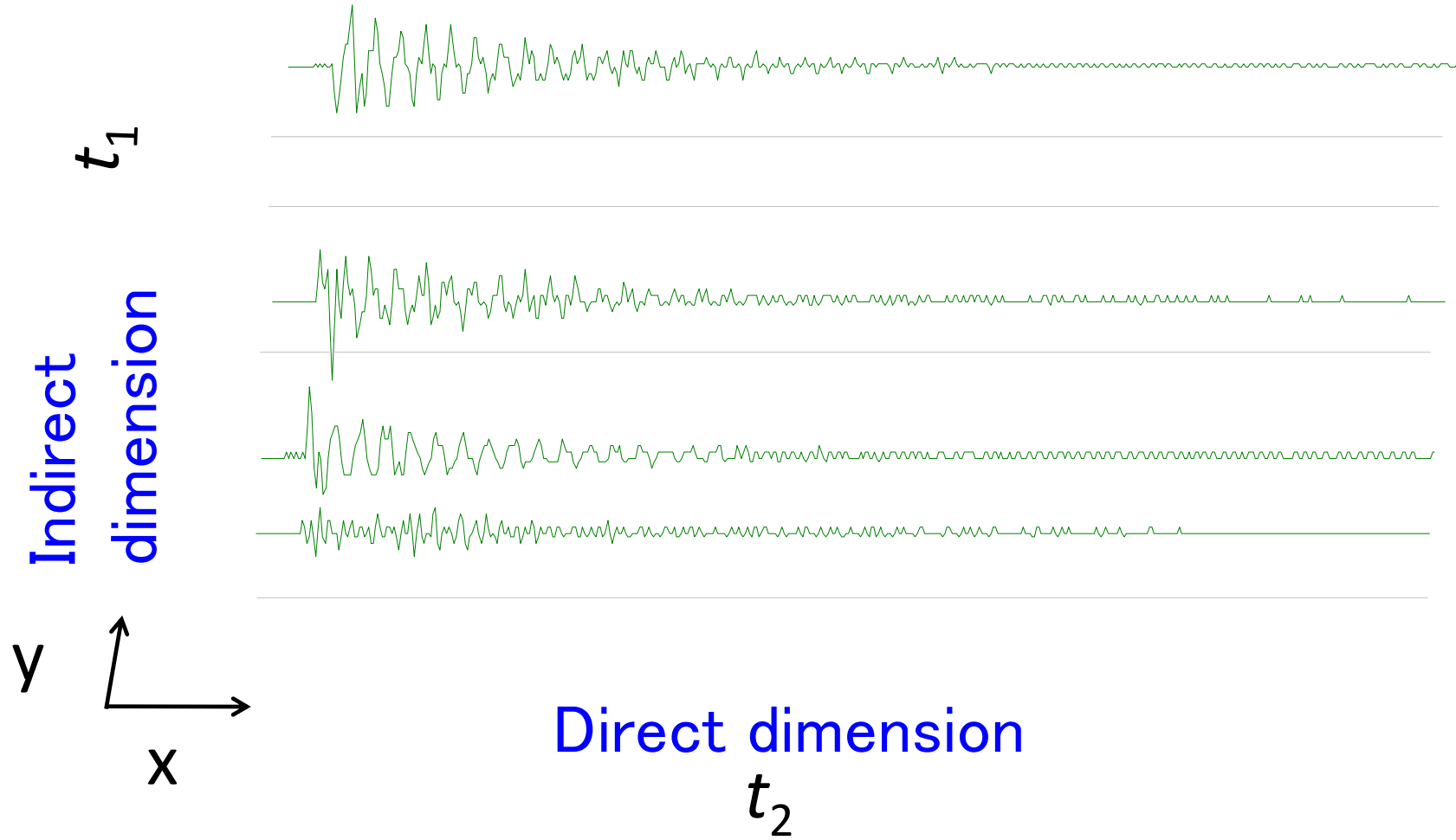
NUS is not bulletproof

Uniform Sampling



FIDs acquired with uniform increments of t_1

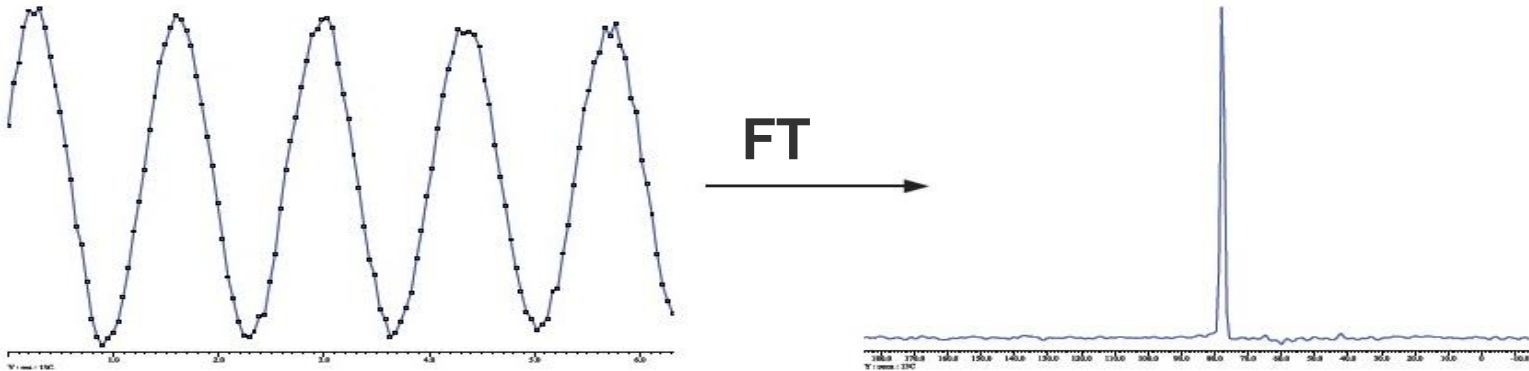
Non-Uniform Sampling



FIDs acquired with non-uniform increments of t_1

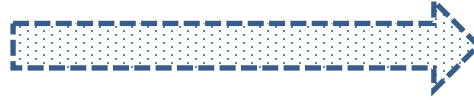
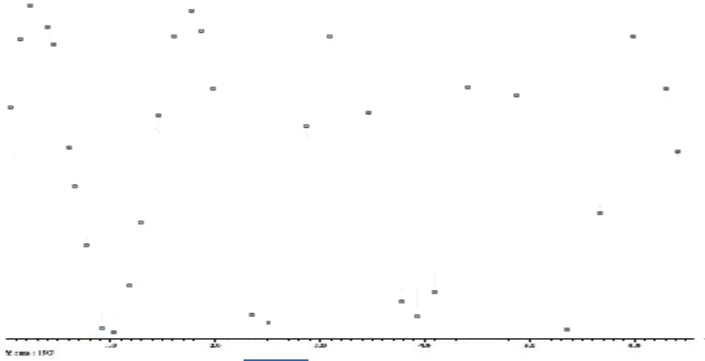
Dealing with non-uniform sampling (indirect dimension)

Uniform sampling

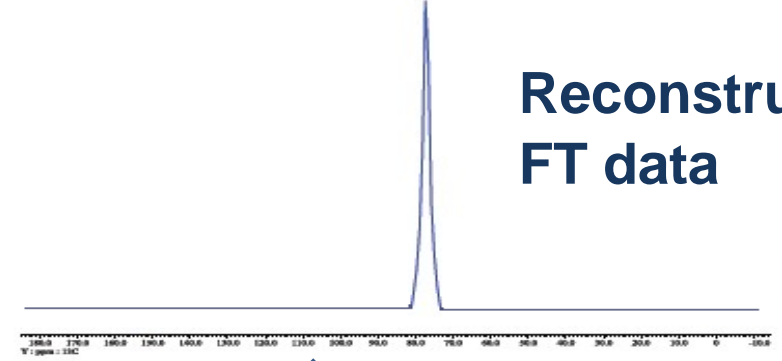


Iterative Soft Thresholding

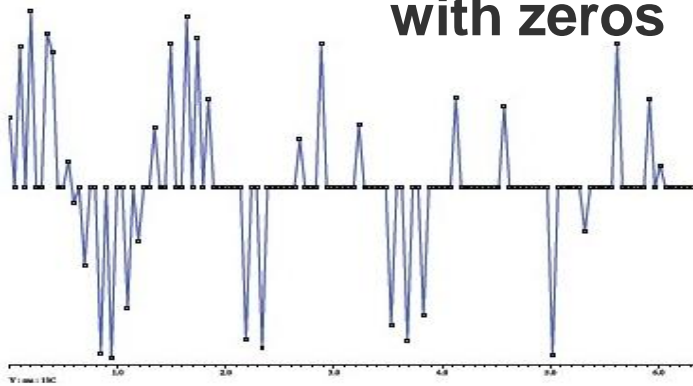
Raw data



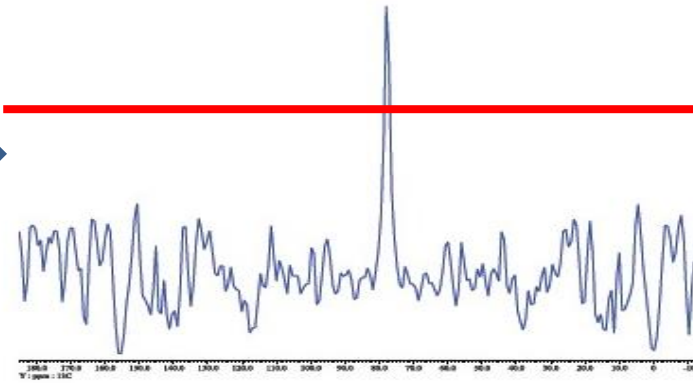
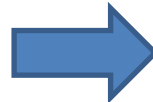
Reconstructed FT data



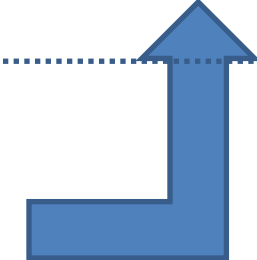
Replace original gaps with zeros



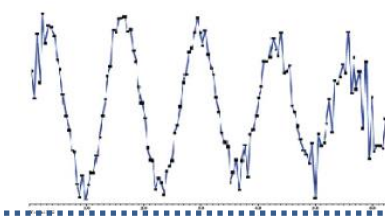
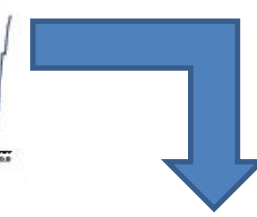
FT



Extract top part and add to final spectrum



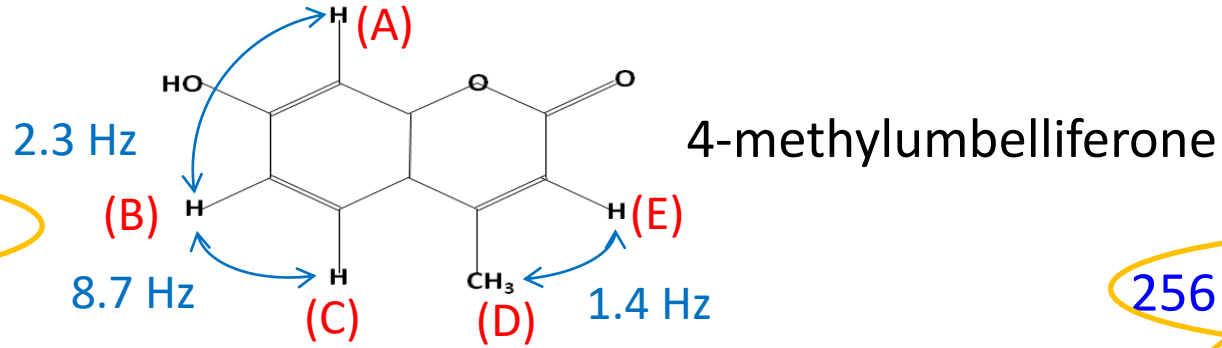
Extract bottom part and do inverse FT



Replace original gaps with zeros



Effect of NUS on COSY

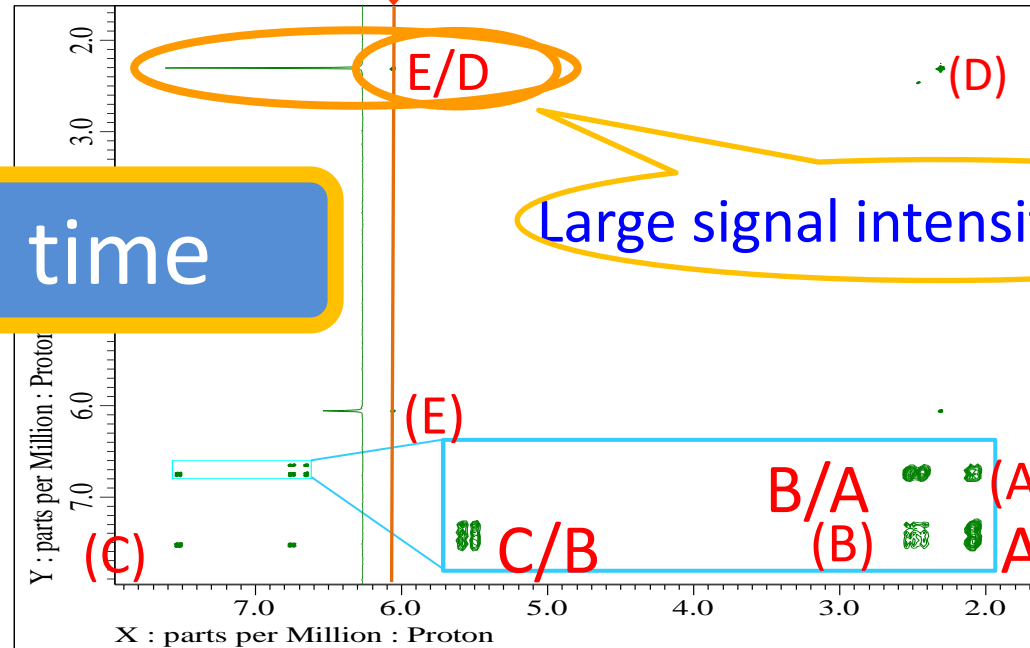
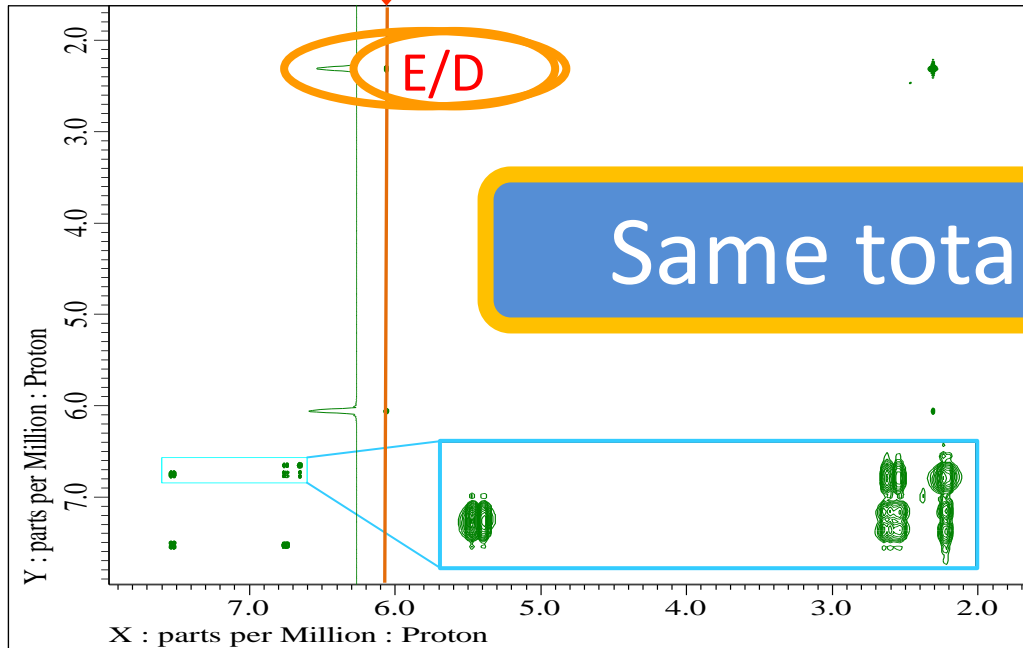


256 sampled points

Uniform ($y_{\text{points}} = 256$)
+ zero fill ($257 \rightarrow 1024$)
+ 2D Fourier transform (FT)

256 sampled points

NUS ($y_{\text{points}} = 1024$, NUS% = 25%)
+ Compressed sensing (CS)



Same total time

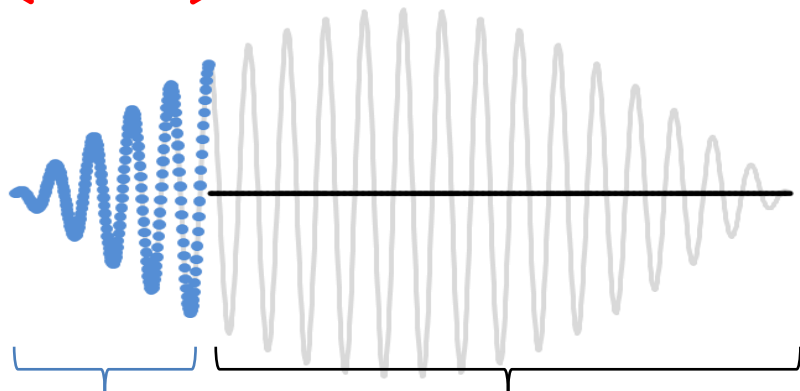
Large signal intensity

COSY signal

Interferogram (t_1 FID) is an echo signal

Uniform sampling + zerofilling

Sampling time

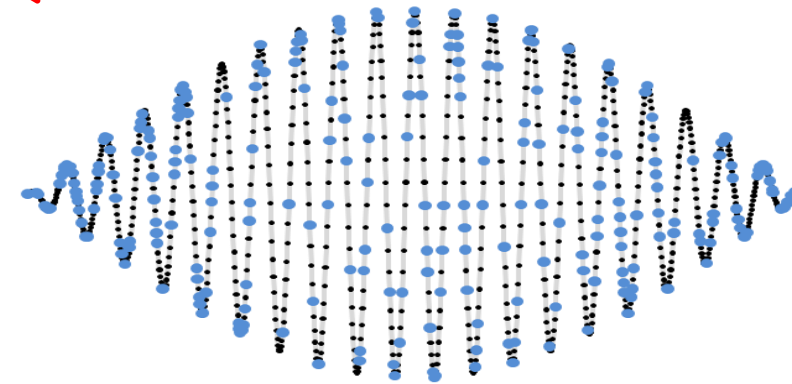


Sampled points

Zerofilled points

NUS + CS

Sampling time



Non-uniformly sampled points

Reconstructed points

Same number of sampled points

NUS allows us to sample the signal in t_1 -domain for longer

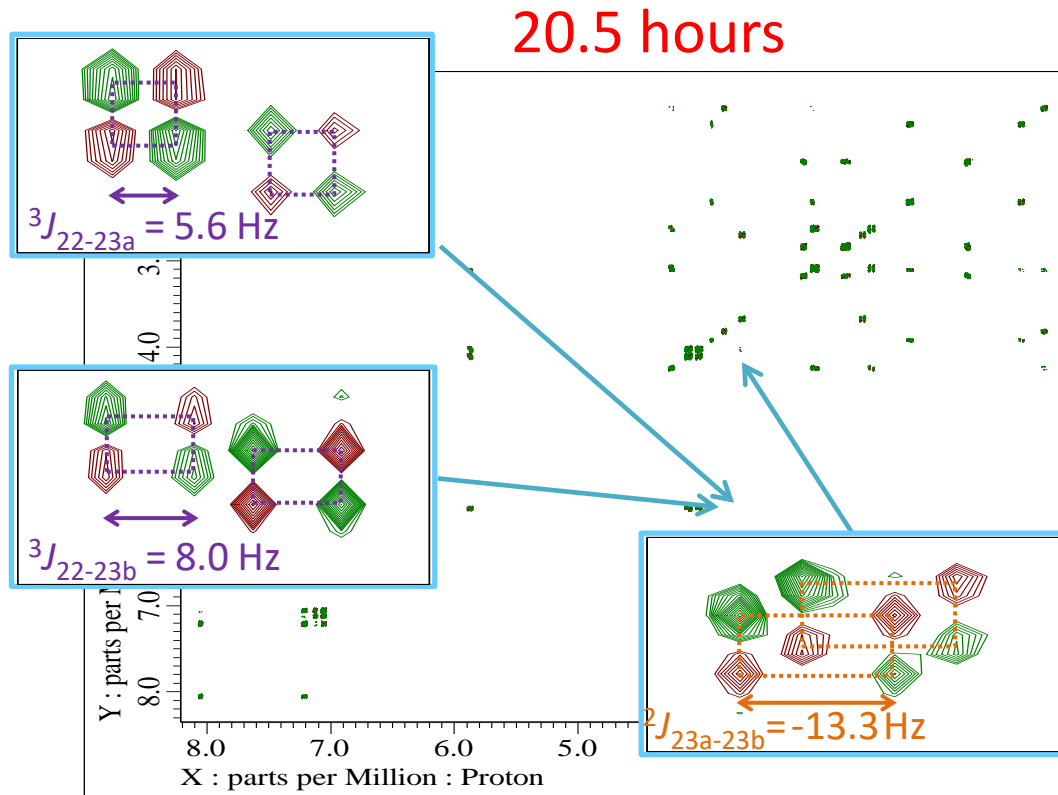
NUS is efficient in COSY, DQF-COSY, E. COSY, etc.

Example of E. COSY (determination of relative sign of J)

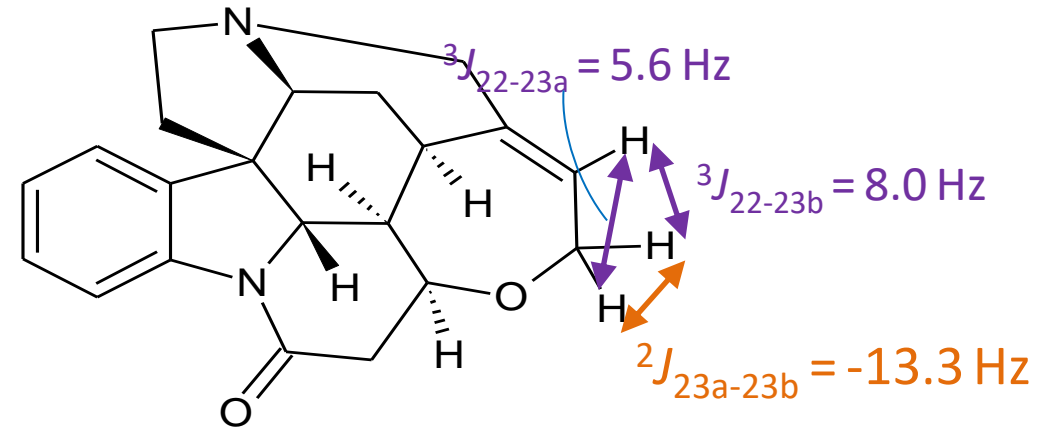
E. COSY requires 24 scans for phase cycling => very long

Even if your sample is concentrated

Even if you have a high-sensitivity NMR probe



100 mM strychnine



In order to measure ${}^1\text{H}$ - ${}^1\text{H}$ coupling constants, high resolution is required

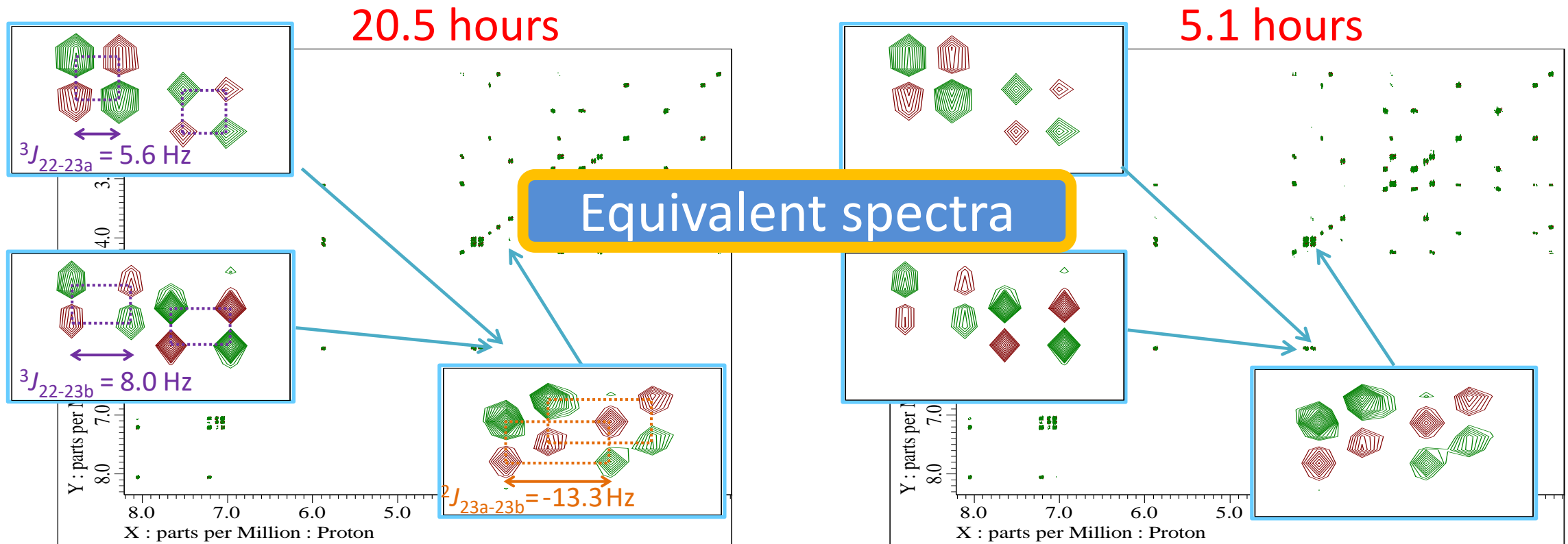
Example of E. COSY

E. COSY requires 24 scans for phase cycling => very long

Even if your sample is concentrated

Even if you have a high-sensitivity NMR probe

Shortened to ¼!



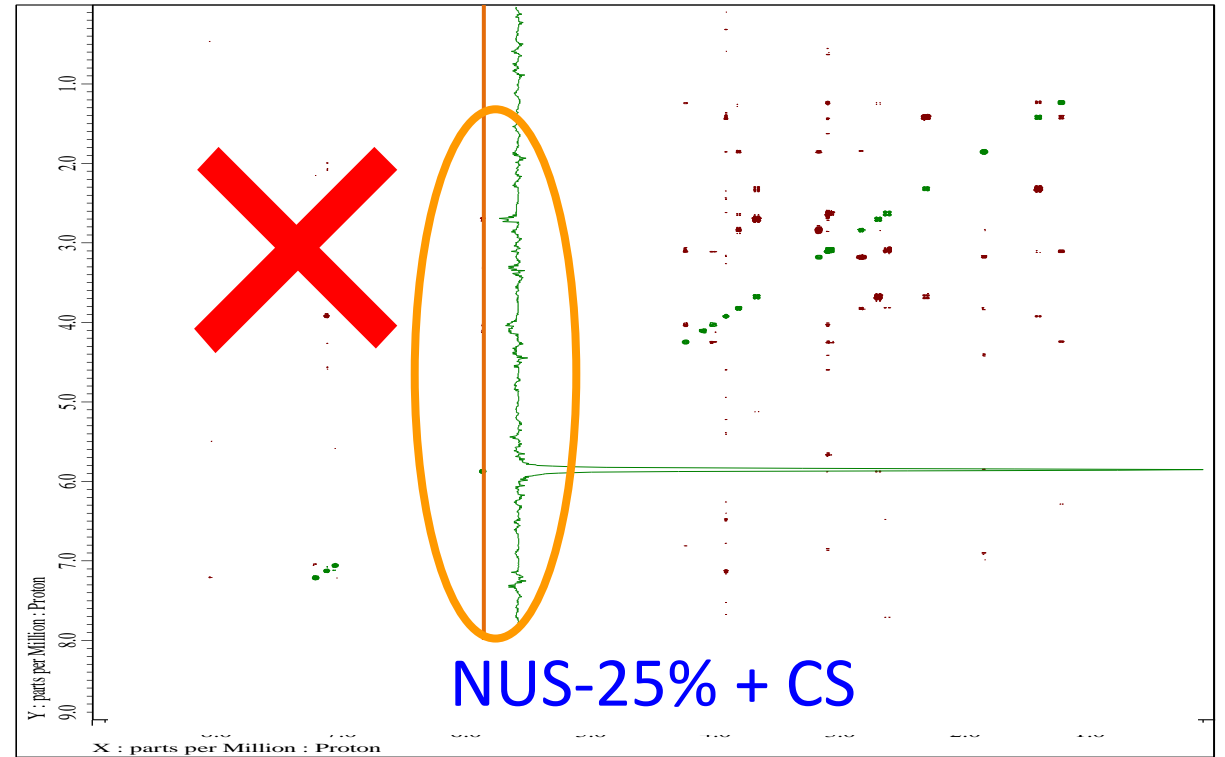
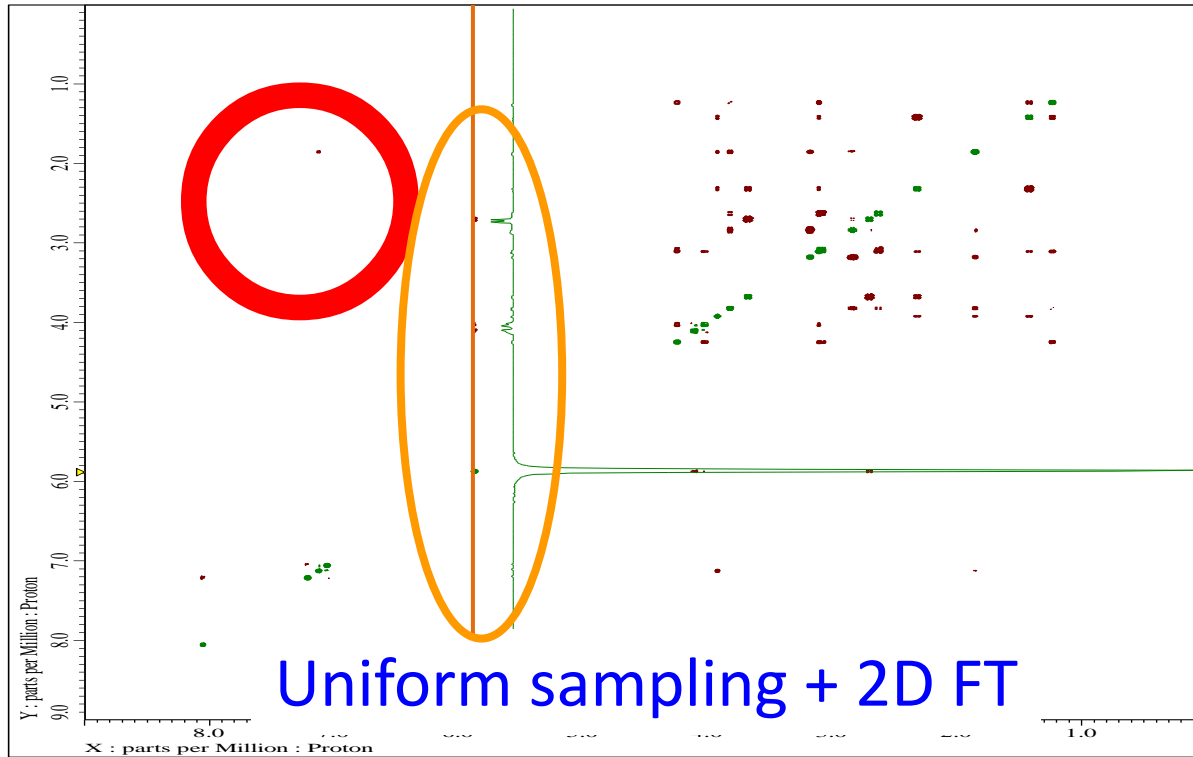
Uniform sampling + 2D FT
512 sampled y_points

25%-NUS + CS
128 sampled y_points

NOESY

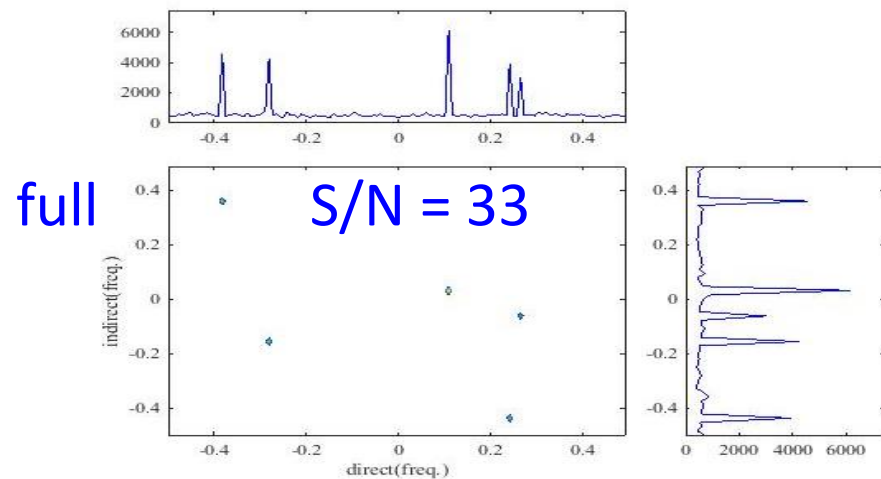
^1H - ^1H NOESY

200 mM strychnine

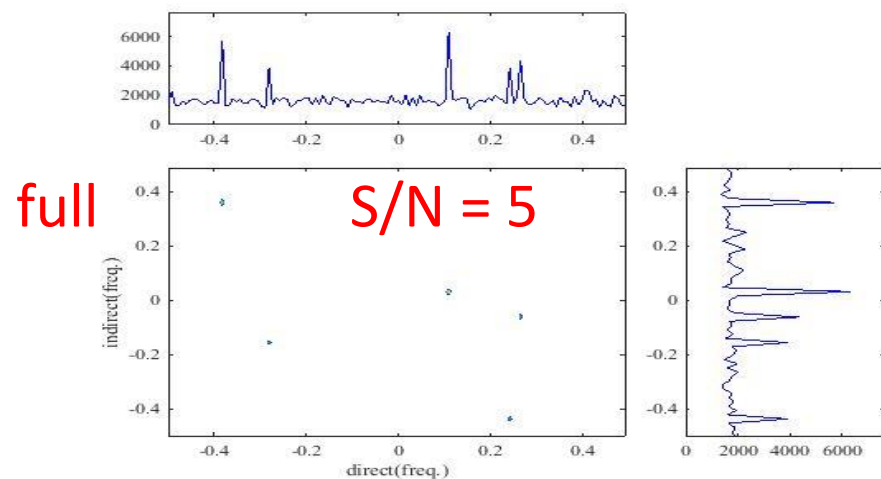
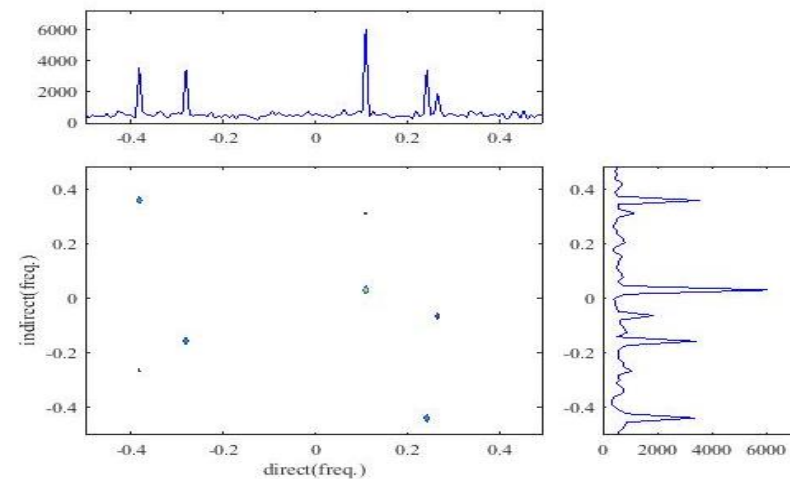


Incomplete reconstruction of low-intensity cross-peaks in the presence of large diagonal peaks

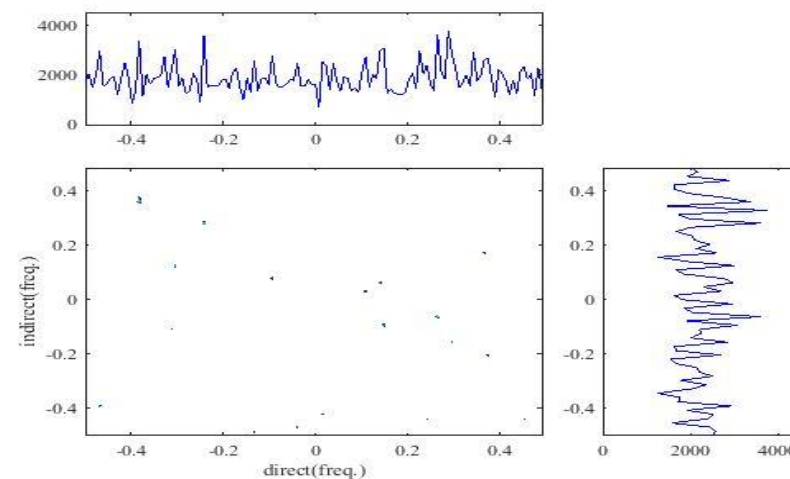
Influence of noise



NUS
(25%)

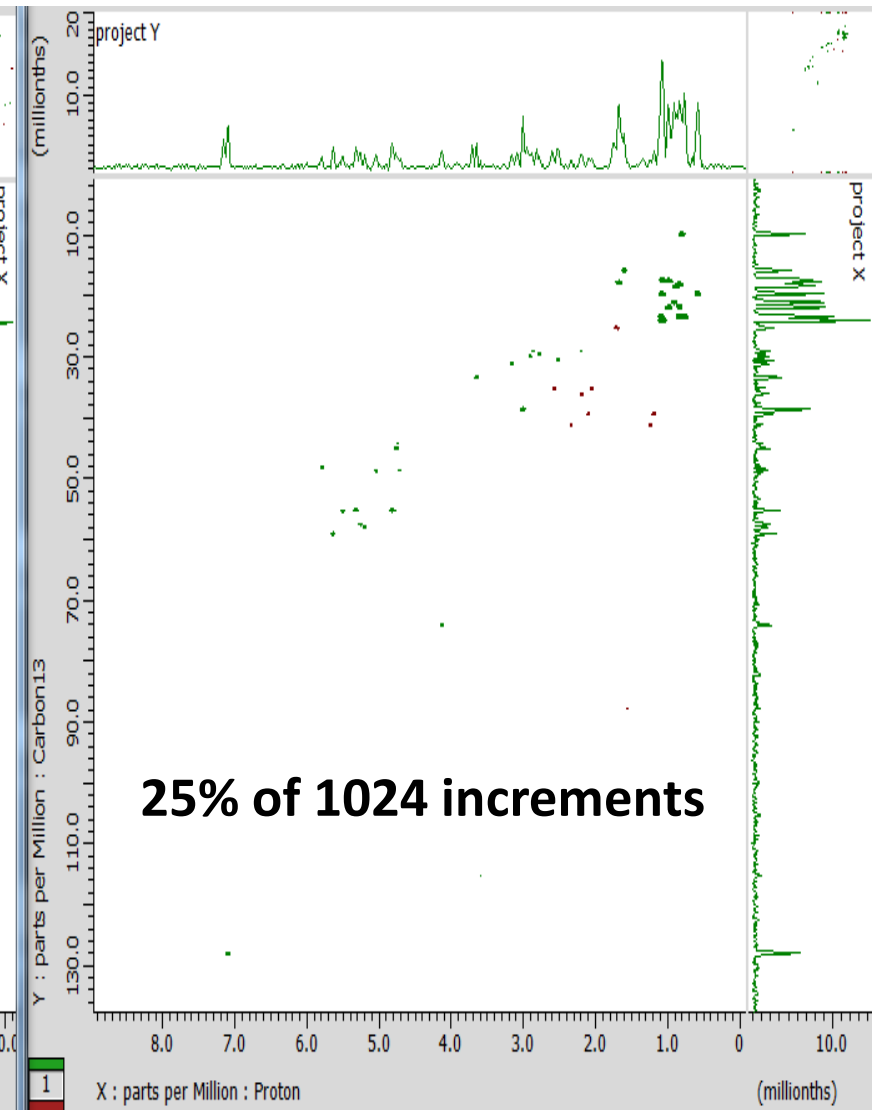
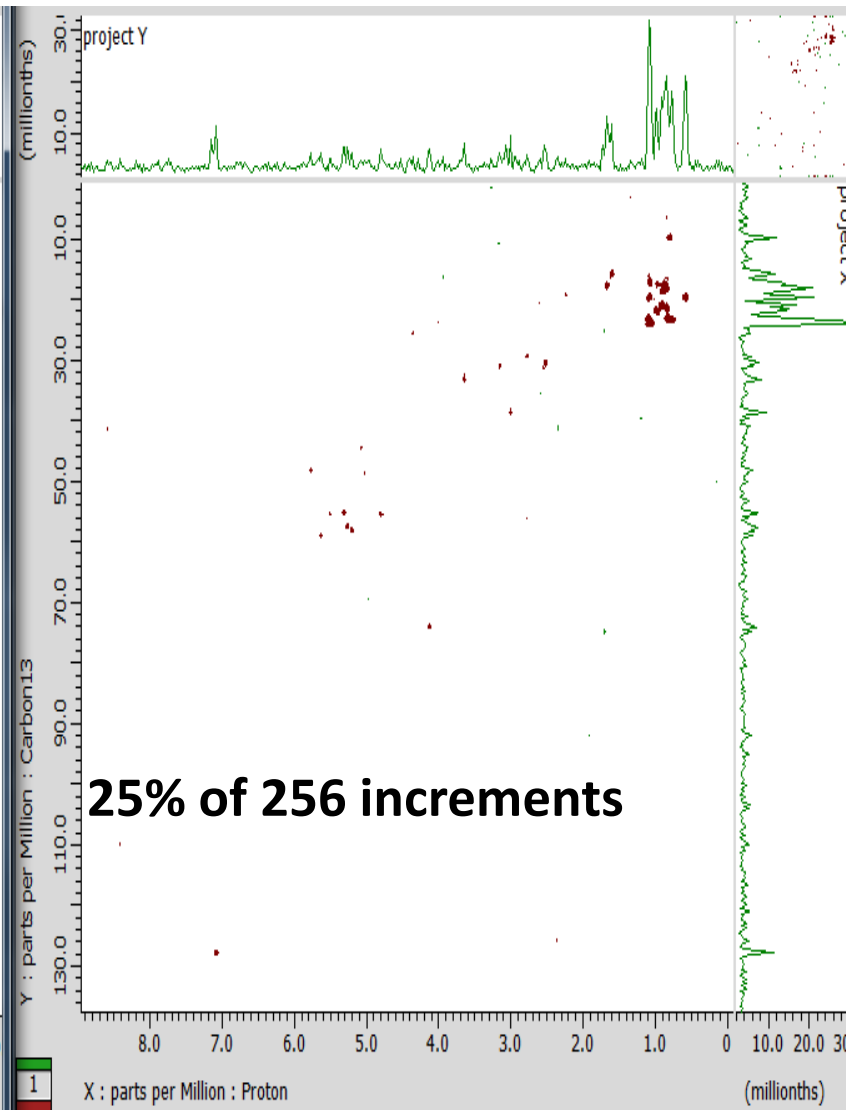
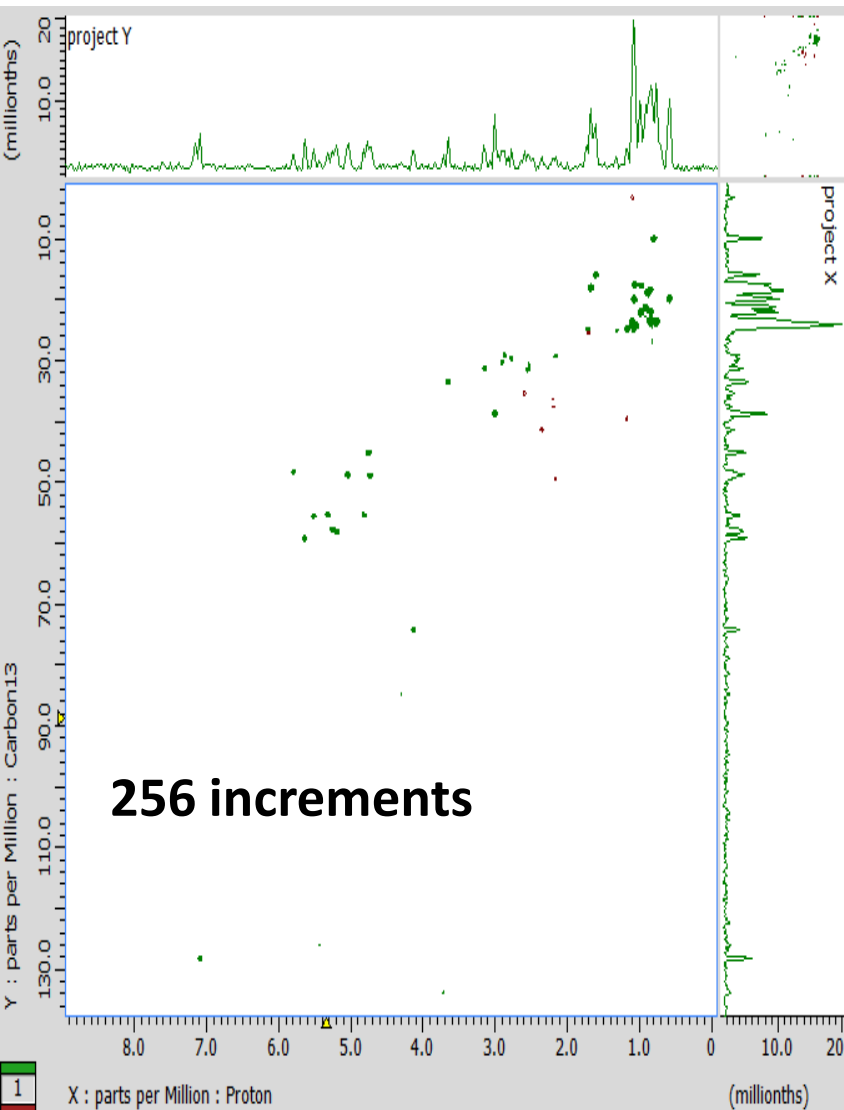


NUS
(25%)



NUS data reconstruction is difficult if S/N is low

HSQC of cyclosporine



So what is the best sampling scheme?

There is no universal solution (just like there is no best pulse flip angle and relaxation delay for pulse acquire experiments)

*Optimal resolution would require sampling up to $\sim 3T_2$, and the S/N reaches a maximum at $\sim 1.2T_2$.**

* Rovnyak, et al. *J Biomol NMR* **30**, 1–10 (2004) <https://doi.org/10.1023/B:JNMR.0000042946.04002.19>

Note:

Large number of increments facilitates observation of small coupling constants correlations, which typically are caused by long range correlations. This is generally desired in HMBC, but not always in COSY.

And how many datapoints are the minimum?

Between 10 and 30 sampling points as many peaks in any column (not peaks but significant peaks, i.e, x tallest peaks), eg, 10 correlations for a proton in an HMBC, then 100-300 sampled points

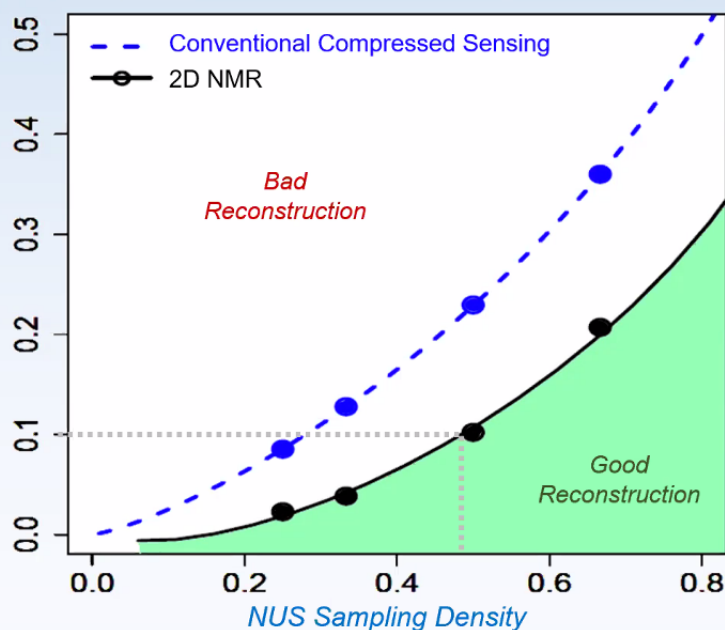
[25% of 256 y_points allows detecting reliably ~2 signals per chemical shift in direct dimension]

For NUS, the Signal Needs to be "Sparse" (More Empty Space Than Signal)
50% Sampling Density Requires ~5 to 10 x more Samples than Signals

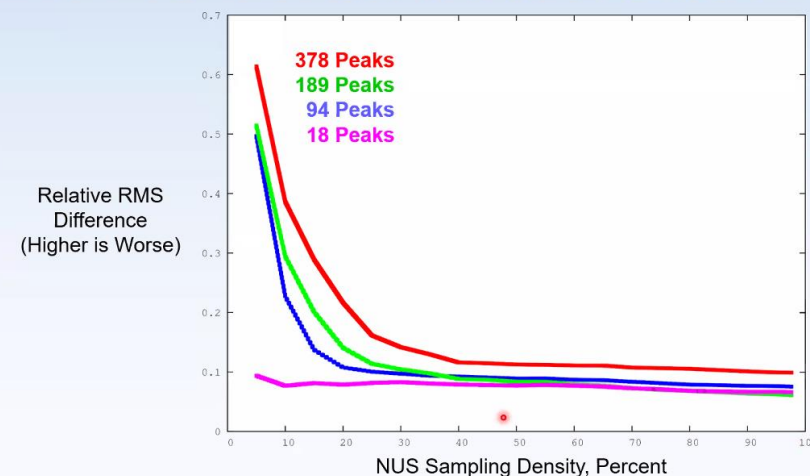
Ratio of Number of Signals to Final Size

For Example:

128 Points Sampled at 50% Density (64 Samples) Allows for ~12 Signals Max per Column



NUS IST Reconstructions Compared to "Ideal" Reconstruction Generated from the Sum of IST Reconstructions of Fully-Sampled Individual Peaks



NIST

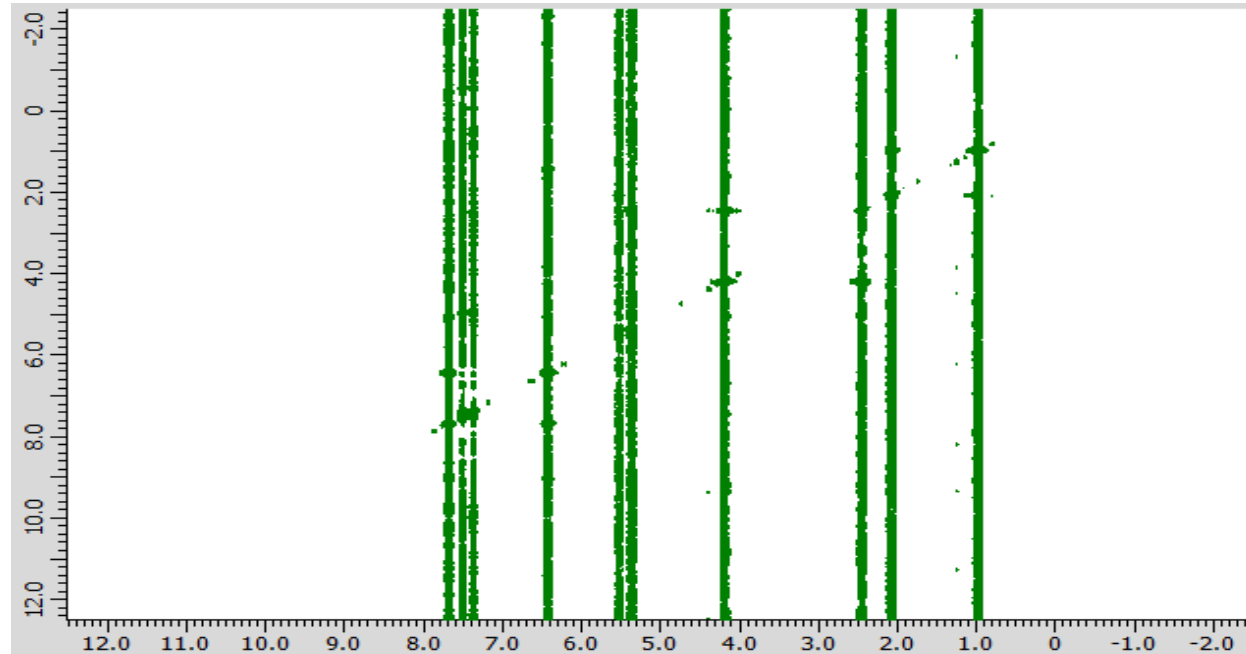
MATERIAL MEASUREMENT LABORATORY

<https://www.youtube.com/watch?v=hK82D-4Flyg>

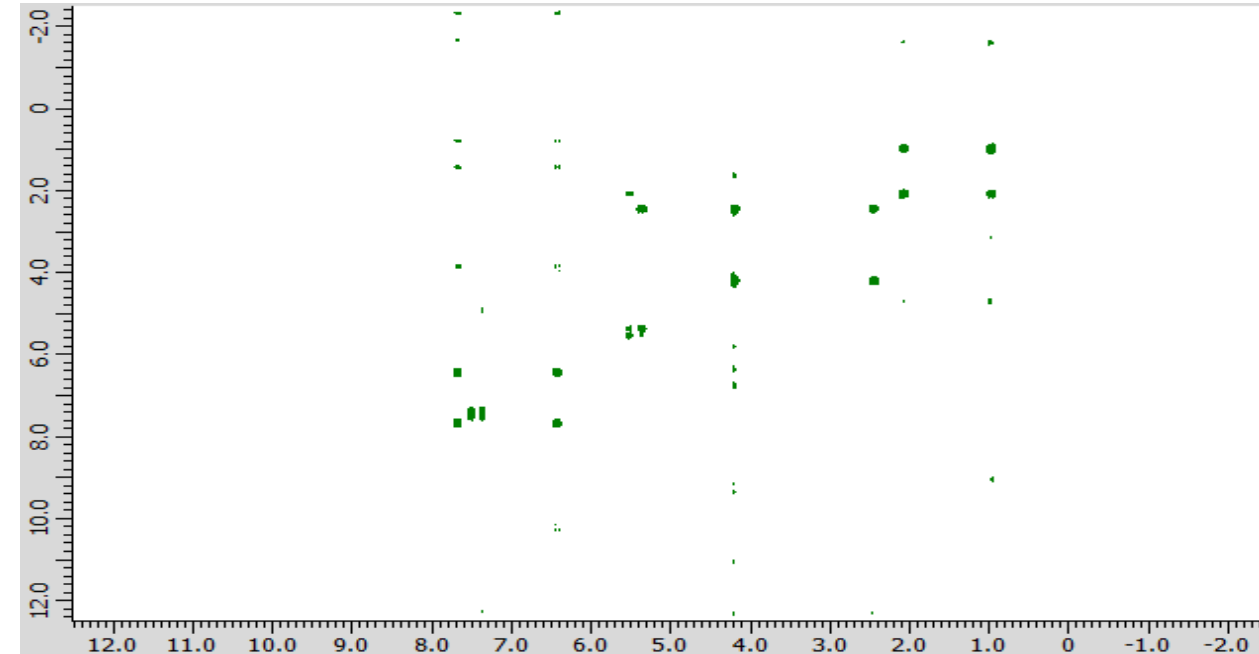
Try other algorithms

COSY 64 increments inflated to 256

- **IST reconstruction**



- **HMS IST reconstruction**

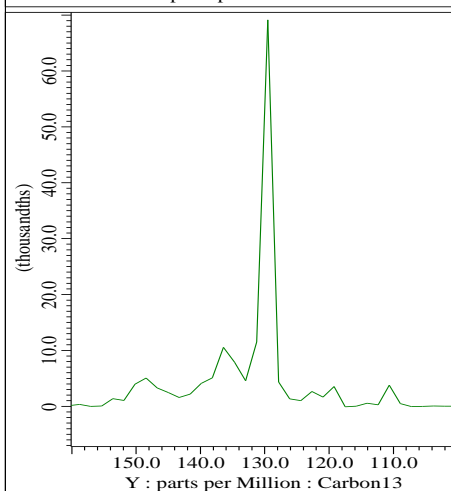
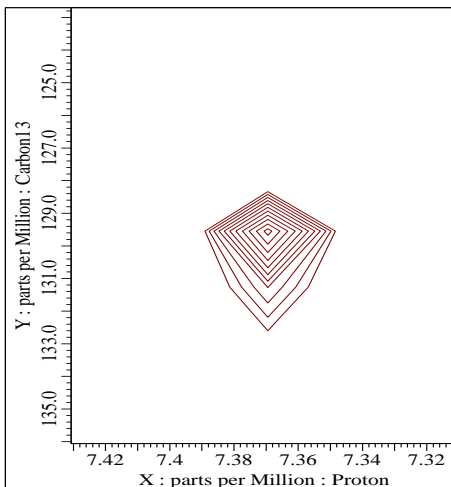


Note that if the experiment was acquired with 256 increments out of 1024, the noise from reconstruction would not be so intense. Sampling density alone does not define how tough is the reconstruction.

Influence of additional zero filling

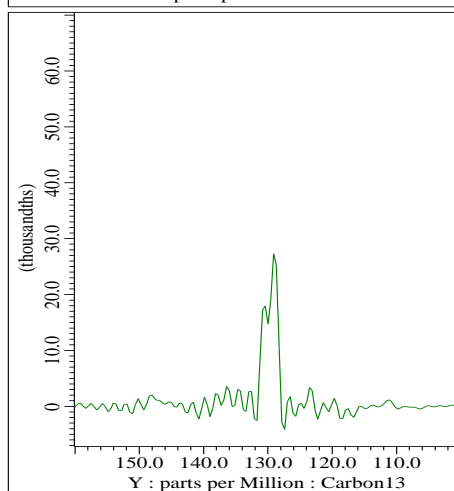
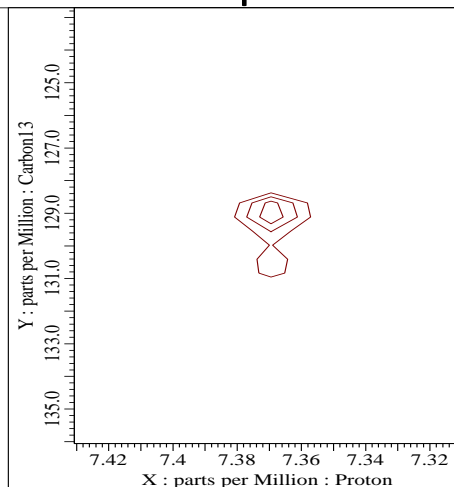
A)

No Zero Filling
IST



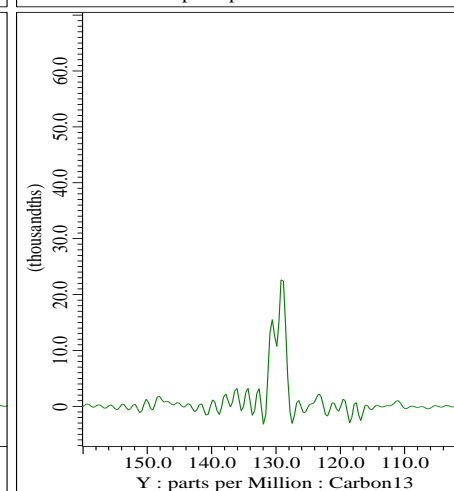
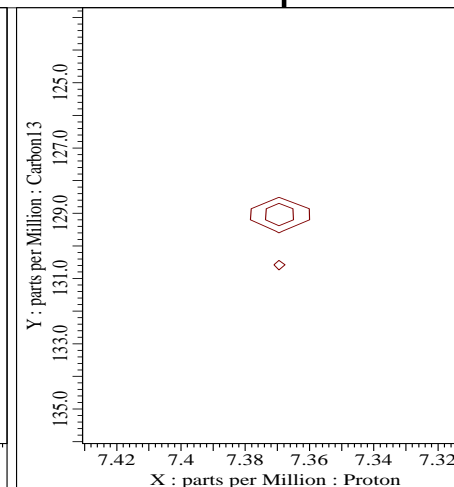
B)

IST
ZF to 512 points



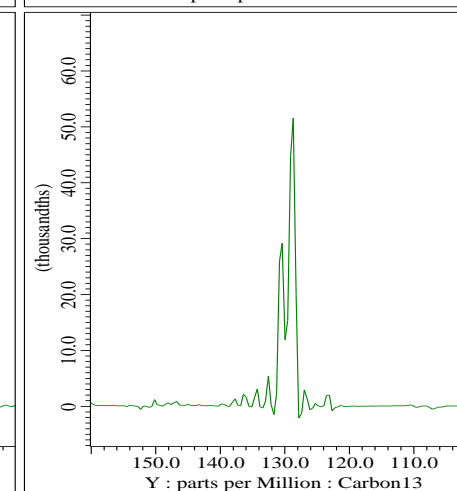
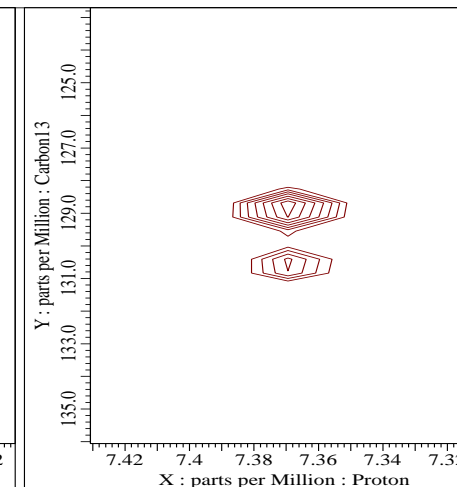
C)

IST
LP to 512 points



D)

ZF to 512 points
IST



2 peaks 1.5ppm apart

Zoomed region of
a me-HSQC
(25% of 128 incr.)

¹³C Projection